Physics II
ISI B.Math
End Semester Exam May 4, 2012

Total Marks: 65
Time: 3 hours

Answer question 1 and ANY THREE from the rest

Question 1. Total Marks: 5x4=20

A body of constant heat capacity C_p and at temperature T_1 is put in thermal contact with a reservoir at temperature T_2 ($T_1 \neq T_2$). Equilibrium between the body and the reservoir is established at constant pressure. Assume that C_p is temperature independent.

a.) Show that $\Delta S_{universe} = C_p f(\frac{T_1}{T_2})$ and determine the function f.
(Hint: Recall: $\Delta S_{universe} = \Delta S_{body} + \Delta S_{reservoir}$)

b.) Show, using the form of the function f, that $\Delta S_{universe} > 0$ regardless of whether T_1 is higher or lower than T_2

c.) Show that $f(\frac{T_1}{T_3}) + f(\frac{T_3}{T_2}) < f(\frac{T_1}{T_2})$ where T_3 is between T_1 and T_2. Explain the implication of this result for the given problem (Hint: you can use the result of Part a.) above.)

d.) Use the result in Part c.) to argue that in principle it is possible to design a process by which a body at T_1 can be heated or cooled to T_2 while $\Delta S_{universe}$ remains zero. What are such processes called in thermodynamics?

Answer ANY THREE from below

Question 2. Total Marks:5+5+3+2

A system whose energy levels are given by $E_n = (n + \frac{1}{2})\hbar\omega$, where n is a non negative integer and $\hbar$ and ω are constants, is in thermal equilibrium with a heat reservoir at temperature T.

a.) Show that the partition function is given by $Z = \frac{e^{-\frac{1}{2}\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}}$ where $\beta = \frac{1}{kT}$.

b.) Show that the average energy $< E >$ is given by $< E >= \hbar\omega[\frac{1}{2} + \frac{1}{e^{\beta\hbar\omega}-1}]$

c.) Determine the limiting values of $< E >$ when $T \to 0$ and when $T \to \infty$.

d.) Interpret both the limiting values obtained in Part c.) above. (Be brief.)

Question 3. Total Marks:3+4+5+3

Consider the Fraunhofer diffraction grating of N slits, each with finite width b, and each slit separated by a distance d. The intensity is given by $I = I_0 \left(\frac{\sin^2 \beta}{\beta^2}\right) \left(\frac{\sin^2 N\gamma}{\gamma^2}\right)$ with usual notation.

a.) Find the locations of principal maxima in terms of d, θ, and λ.

b.) Find the locations of the minima between two principal maxima, and show that there are $N - 2$ secondary maxima.

c.) Now consider diffraction pattern formed by two wavelengths λ and $\lambda + \Delta\lambda$. These wavelengths are said to be well resolved when the principal maxima $\lambda + \Delta\lambda$ falls on the first minimum of λ.Show that this happens when the following condition is satisfied $\frac{\lambda}{\Delta\lambda} = mN$.

d.) For the D1 and D2 lines of sodium ($\lambda = 5890$ Angstrom and $\lambda + \Delta\lambda = 5896$ Angstrom, what should be the number of slits for the lines to be well resolved in the first order.)

Question 4. Total Marks:3+4+4+4

Consider a spring that obeys Hooke's law, namely that at constant temperature T the tension X in the spring is proportional to the amount of its elongation x. In other words $X = kx$ where k is independent of x but can depend on T. Ignore thermal expansion of the spring

a.) Write the differential expression dF for Helmholtz Free energy F for this system in terms of dT and dx.

b.) Derive the following equation: $F(T, x) = F(T, 0) + \frac{1}{2}kx^2$.

c.) Derive the corresponding equation for the entropy $S(T, x)$.

d.) Show that the expression of internal energy U of the spring is given by

$U(T, x) = U(T, 0) + \frac{1}{2}(k - T\frac{dk}{dT})x^2$.

(Hint: Use the equivalent Maxwell Relations appropriate for this system. Write F and S in terms of derivatives of thermodynamic potentials and integrate these.)

Question 5. Total Marks:5+6+4

In thermodynamics the Joule Thomson effect describes the temperature change of a gas when it is forced through a valve or porous plug while kept insulated so that no heat is exchanged with the environment.

a.) Show that the enthalpy H remains constant in the process.

b.) Starting with the equation $dH = TdS + Vdp$, show that

$\mu = \frac{V}{C_p}(\alpha T - 1)$ where $\mu = \left(\frac{\partial T}{\partial P}\right)_H$, and $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$

c.) Show that for a gas whose equation of state is
$PV = RT + BP + CP^2$ where B and C are functions of temperature only,
the pressure at the inversion curve (given by $\mu = 0$) is determined by the equation

$$P = \frac{T\frac{dB}{dT} - B}{C - T\frac{dC}{dT}}.$$

Useful Information

Thermodynamic Identities / Equations

$dQ = dU + dW$ where dW is the work done BY the system

For a typical PVT system,

$H = U - PV$,

$F = U - TS$,

$G = U - TS + PV$.

Maxwell's Relations:
$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$
$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$
$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$
$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$